RIGID TRANSFORMATIONS 6.1.1 – 6.1.4

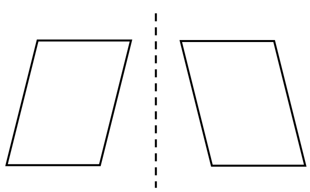
Studying transformations of geometric shapes builds a foundation for a key idea in geometry: congruence. In this introduction to transformations, the students explore three rigid motions: translations, reflections, and rotations. A translation slides a figure horizontally, vertically or both. A reflection flips a figure across a fixed line (for example, the x-axis). A rotation turns an object about a point (for example, $(0, 0)$). This exploration is done with simple tools that can be found at home (tracing paper) as well as with computer software. Students change the position and/or orientation of a shape by applying one or more of these motions to the original figure to create its image in a new position without changing its size or shape. Transformations also lead directly to studying symmetry in shapes. These ideas will help with describing and classifying geometric shapes later in the course.

For additional information, see the Math Notes box in Lesson 6.1.3 of the *Core Connections, Course 3* text.

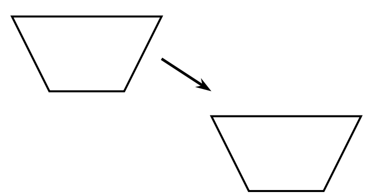
Example 1

Decide which transformation was used on each pair of shapes below. Some may be a combination of transformations.

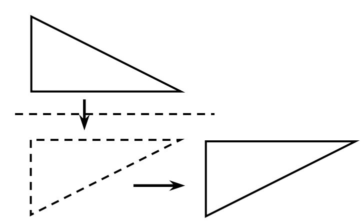
a.



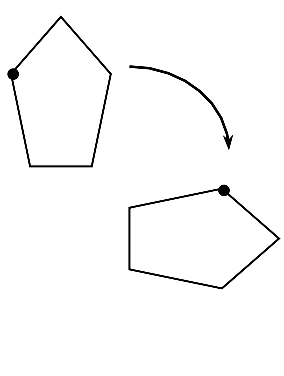
b.



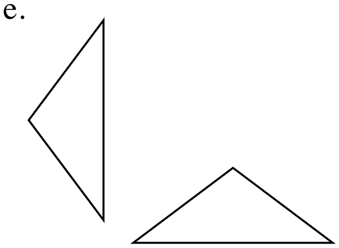
c.



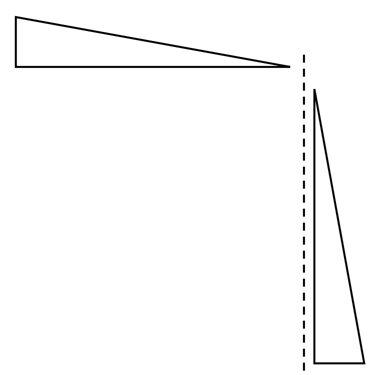
d.



e.



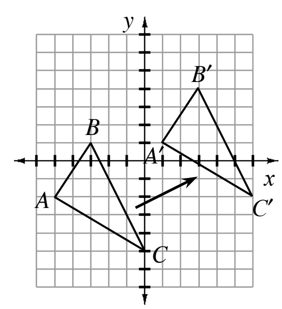
f.



Identifying a single transformation is usually easy for students. In part (a), the parallelogram is reflected (flipped) across an invisible vertical line. (Imagine a mirror running vertically between the two figures. One figure would be the reflection of the other.) Reflecting a shape once changes its orientation, that is, how its parts “sit” on the flat surface. For example, in part (a), the two sides of the figure at left slant upwards to the right, whereas in its reflection at right, they slant upwards to the left. Likewise, the angles in the figure at left “switch positions” in the figure at right.

In part (b), the shape is translated (or slid) to the right and down. The orientation is the same. Part (c) shows a combination of transformations. First the triangle is reflected (flipped) across an invisible horizontal line. Then it is translated (slid) to the right. The pentagon in part (d) has been rotated (turned) clockwise to create the second figure. Imagine tracing the first figure on tracing paper, then holding the tracing paper with a pin at one point below the first pentagon, then turning the paper to the right (that is, clockwise) 90º. The second pentagon would be the result. Some students might see this as a reflection across a diagonal line. The pentagon itself could be, but with the added dot, the entire shape cannot be a reflection. If it had been reflected, the dot would have to be on the corner below the one shown in the rotated figure. The triangles in part (e) are rotations of each other (90º clockwise again). Part (f) shows another combination. The triangle is rotated (the horizontal side becomes vertical) but also reflected since the longest side of the triangle points in the opposite direction from the first figure.

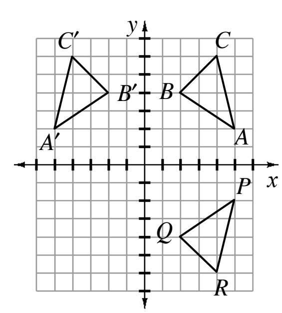
Example 2



Translate (slide) ΔABC right six units and up three units. Give the coordinates of the new triangle.

The original vertices are $A(-5, -2)$, $B(-3, 1)$, and $C(0, -5)$. The new vertices are $A'(1, 1)$, $B'(3, 4)$, and $C'(6, -2)$. Notice that the change to each original point (x, y) can be represented by $(x + 6, y + 3)$.

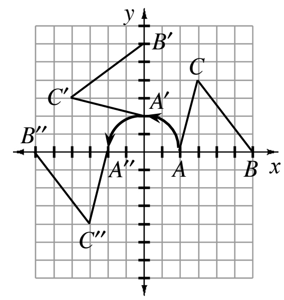
Example 3



Reflect (flip) ΔABC with coordinates $A(5, 2)$, $B(2, 4)$, and $C(4, 6)$ across the y-axis to get $\Delta A'B'C'$. The key is that the reflection is the same distance from the y-axis as the original figure. The new points are $A'(-5, 2)$, $B'(-2, 4)$, and $C'(-4, 6)$. Notice that in reflecting across the y-axis, the change to each original point (x, y) can be represented by $(-x, y)$.

If you reflect ΔABC across the x-axis to get ΔPQR, then the new points are $P(5, -2)$, $Q(2, -4)$, and $R(4, -6)$. In this case, reflecting across the x-axis, the change to each original point (x, y) can be represented by $(x, -y)$.

Example 4



Rotate (turn) ΔABC with coordinates $A(2, 0)$, $B(6, 0)$, and $C(3, 4)$ 90° counterclockwise about the origin $(0, 0)$ to get $\Delta A'B'C'$ with coordinates $A'(0, 2)$, $B'(0, 6)$, and $C'(-4, 3)$. Notice that for this 90° counterclockwise rotation about the origin, the change to each original point (x, y) can be represented by $(-y, x)$.

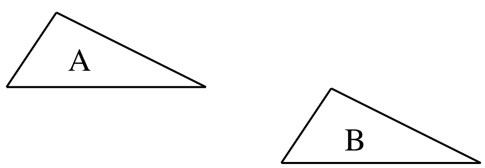
Rotating another 90° (180° from the starting location) yields $\Delta A''B''C''$ with coordinates $A''(-2, 0)$, $B''(-6, 0)$, and $C''(-3, -4)$.

For this 180° counterclockwise rotation about the origin, the change to each original point (x, y) can be represented by $(-x, -y)$. Similarly a 270° counterclockwise or 90° clockwise rotation about the origin takes each original point (x, y) to the point $(y, -x)$.

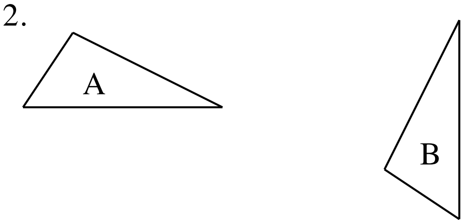
Problems

For each pair of triangles, describe the transformation that moves triangle A to the location of triangle B.

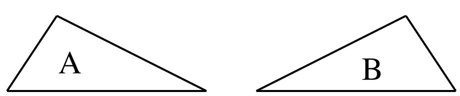
1.



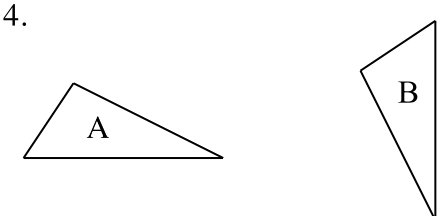
2.



3.



4.



For the following problems, refer to the figures below:

Figure A

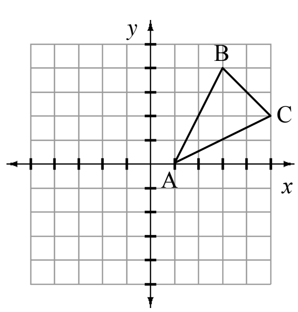


Figure B

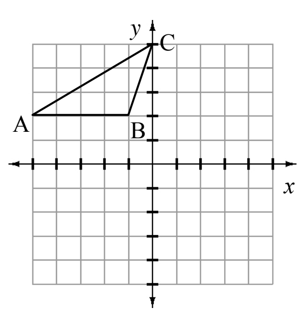
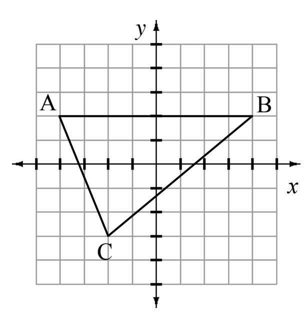


Figure C



State the new coordinates after each transformation.

5. Slide figure A left 2 units and down 3 units.

6. Slide figure B right 3 units and down 5 units.

7. Slide figure C left 1 unit and up 2 units.

8. Flip figure A across the x-axis.

9. Flip figure B across the x-axis.

10. Flip figure C across the x-axis.

11. Flip figure A across the y-axis.

12. Flip figure B across the y-axis.

13. Flip figure C across the y-axis.

14. Rotate figure A 90º counterclockwise about the origin.

15. Rotate figure B 90º counterclockwise about the origin.

16. Rotate figure C 90º counterclockwise about the origin.

17. Rotate figure A 180º counterclockwise about the origin.

18. Rotate figure C 180º counterclockwise about the origin.

19. Rotate figure B 270º counterclockwise about the origin.

20. Rotate figure C 90º clockwise about the origin.

Answers (1 to 4 may vary; 5 to 20 given in the order A′, B′, C′)

1. translation
2. rotation and translation
3. reflection
4. rotation and translation
5. (–1, –3), (1, 2), (3, –1)
6. (–2, –3), (2, –3), (3, 0)
7. (–5, 4), (3, 4), (–3, –1)
8. (1, 0), (3, –4), (5, –2)
9. (–5, –2), (–1, –2), (0, –5)
10. (–4, –2), (4, –2), (–2, 3)
11. (–1, 0), (–3, 4), (–5, 2)
12. (5, 2), (1, 2), (0, 5)
13. (4, 2), (–4, 2), (2, –3)
14. (0, 1), (–4, 3), (–2, 5)
15. (–2, –5), (–5, 0), (–2, –1)
16. (–2, –4), (–2, 4), (3, –2)
17. (–1, 0), (–3, –4), (–5, –2)
18. (4, –2), (–4, –2), (2, 3)
19. (2, 5), (2, 1), (5, 0)
20. (2, 4), (2, –4), (–3, 2)

